Hypothesis Testing
&
One sample t-test

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LOGO

Concepts

◆**A hypothesis is a statement(or claim) that may or may not be true**

1. - Average blood pressure is $\mu = 13.5$
2. - The effect of drug 2 in better than drug1

- A hypothesis can be tested using statistical tools to determine whether it can be rejected

- 2 possible outcomes of hypothesis testing:
  - (1) The hypothesis can be rejected
  - (2) The hypothesis cannot be rejected

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Concepts

- A hypothesis is a statement that may or may not be true (about population **parameter**)
  - A hypothesis can be tested using statistical tools by **statistics** to determine whether it can be rejected
  - 2 possible outcomes of **hypothesis testing**:
    - (1) The hypothesis can be rejected
    - (2) The hypothesis cannot be rejected

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Concepts

- **Null hypothesis (H0):** the hypothesis to be tested using statistical tools.

- **Alternative Hypothesis (Ha):** an alternative to H0 that the researcher believes to be true

- H0 is the negation of the Ha
  - If the researcher can reject $\boldsymbol{H_0}$, this suggests that $\boldsymbol{H_a}$ might be true

Concepts: Hypothesis Test Errors

- **A Type I Error for a statistical test is the error made by rejecting the null hypothesis when it is true**
  - probability of a Type I Error is **α**
- **A Type II Error for a statistical test is the error made by accepting the null hypothesis when it is false**
  - probability of a Type II Error is **β**

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Concepts: Hypothesis Test Errors

Your Statistical Decision	True state of null hypothesis	
	H0 True	H0 False
Reject H0	Type I error (α)	Correct
Do not reject H0	Correct	Type II Error (β)

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Concepts: *p* - Value

- α is the probability of making a type I error, and is sometimes called the significance level of a statistical test

- The smallest value of α for which you can still reject the null hypothesis is the *p* – value of the hypothesis test

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Concepts: Confidence Interval

- A confidence interval gives an estimated range of values which is likely to include an unknown population parameter.

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Concepts: ??

Example: The effect of drug 2 in better than drug1

Say what is the . . . .

Type I Error and α??

Type II Error and β??

- Most important Error??

Power of the test??

P-value??

Confidence Interval for mean??

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Concepts: Two Tailed Hypothesis Testing

- **Two-tailed test of hypothesis: used if you want to test whether the population parameter of interest (“θ”) is different than some hypothesized value θ_0 (in either direction)**

$$H_0: \theta = \theta_0$$
$$H_a: \theta \neq \theta_0$$

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Concepts: One Tailed Hypothesis Testing

- **One-tailed test of hypothesis: used if you want to test whether the population parameter of interest ("θ") is different than some hypothesized value θ_0 in a particular direction**

Left tailed test:

$$H_0: \theta = \theta_0$$
$$H_a: \theta < \theta_0$$

Right tailed test:

$$H_0: \theta = \theta_0$$
$$H_a: \theta > \theta_0$$

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Concepts: Critical Value

- The Test Statistics will tell you whether or not you can reject the null hypothesis. The value of the test statistics will fall in 1 of 2 regions:
  - The **Acceptance Region** includes all values of the test statistic that do not contradict the null hypothesis
  - The **Rejection Region** includes all values of the test statistics that contradict the null hypothesis
- The **Critical Value** of the test statistic separates the acceptance region and the rejection region

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Approaches of Testing Hypothesis

- Generally, there are three approaches for testing hypothesizes in statistical packages:

1. Critical value approach
2. P-value approach
3. Confidence interval

Two Tailed Hypothesis Test of Population Mean (One sample t-test)

- **Suppose we want to know whether the mean of a population is different than some value μ_0**

$$H_0 : \mu = \mu_0$$
$$H_a : \mu \neq \mu_0$$

- **Our best estimate of μ is x-bar (the sample mean). What values of x-bar would lead us to believe that H_0 is false?**
  - If we observe a value of x-bar that is “too” large or small (i.e. too many standard deviations from the mean) we can reject H_0

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Two Tailed Hypothesis Test of Population Mean

- We know by **CLT** that the sampling distribution of x-bar is approximately **normal** when n is large.

- If H_0 is true, the number of standard deviations that x-bar lies from its mean can be computed using the z score:

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \qquad t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

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One sample t-test

1) Hypothesis: $H_0 : \mu = \mu_0$

$$H_a : \mu \neq \mu_0$$

2) Test statistic: small sample case

$$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

3) Critical value, rejection and acceptance region:

- The bigger the absolute value of t is, the more possible to reject null hypothesis.

- The critical value depends on significance level

- rejection region: $|t| > t_{\alpha/2}$ d.f.=n-1

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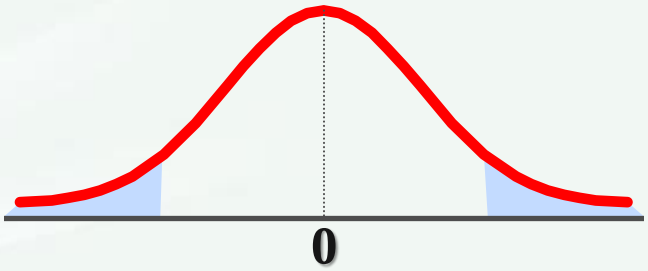
Given the sample information:

H_0: $\mu = 12$

H_1: $\mu \neq 12$

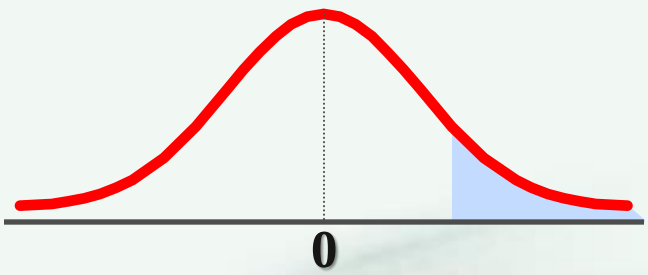
Two-tail test



H_0: $\mu \leq 12$

H_1: $\mu > 12$

Upper-t`ail test



H_0: $\mu \geq 12$

H_1: $\mu < 12$

Rejection Region or Critical Value Approach:

Level of significance = α

$H_0: \mu = 12$

$H_1: \mu \neq 12$

Two-tail test

$H_0: \mu \leq 12$

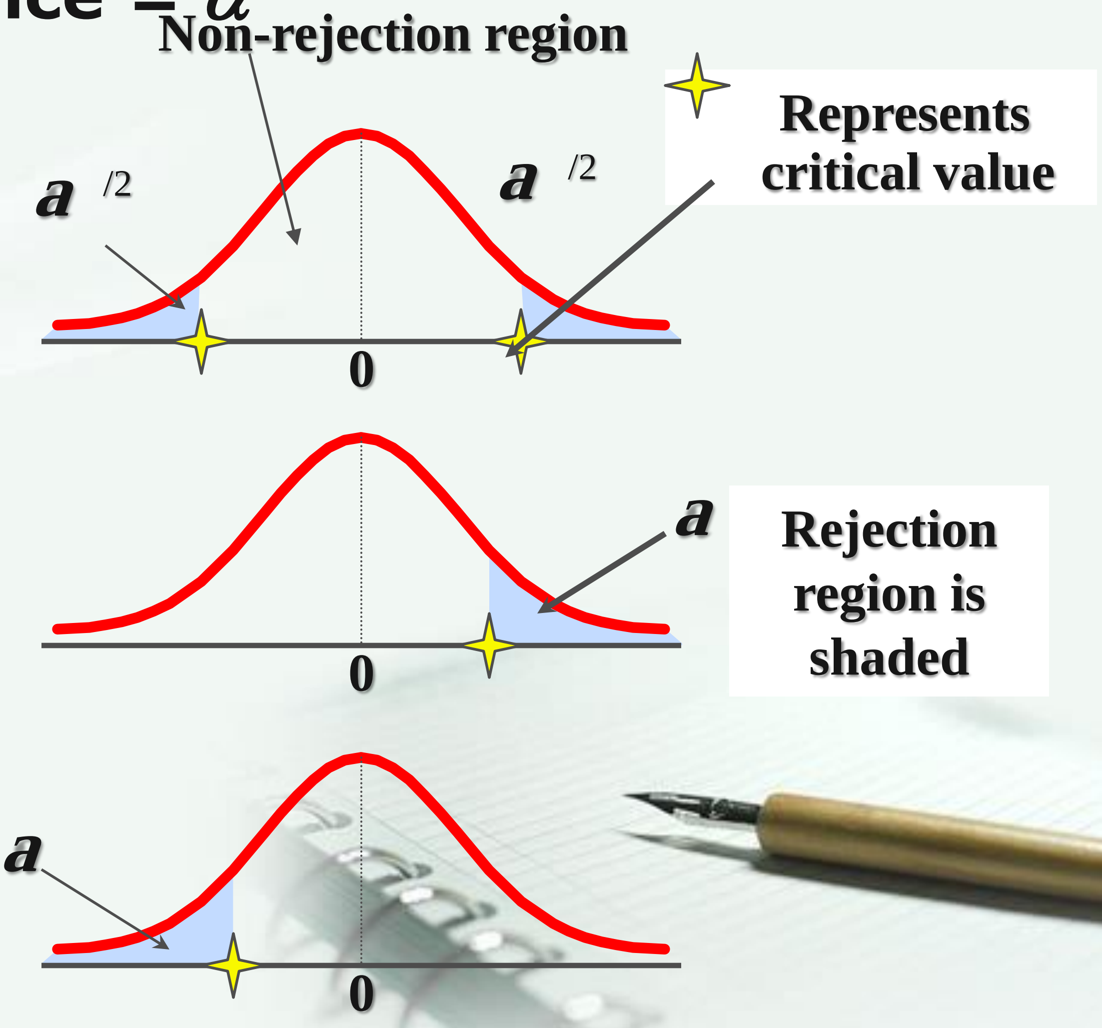
$H_1: \mu > 12$

Upper-tail test

$H_0: \mu \geq 12$

$H_1: \mu < 12$

Lower-tail test



Article 1

Facial profile preferences of black women before and after orthodontic treatment

Javonne McKoy-White,[a] Carla A. Evans,[b] Grace Viana,[c] Nina K. Anderson,[d] and Donald B. Giddon[e]
Fairburn, Ga, Chicago, Ill, and Boston, Mass

Introduction: The purposes of this study were to determine (1) profile preferences of black female patients and (2) whether they can recognize their own profile images before and after orthodontic treatment. **Methods:** Fifteen black orthodontists, 15 white orthodontists, and 15 black female patients were asked to indicate the images they considered most pleasing and to determine a zone of acceptability for 3 black female profiles. Raters used the PERCEPTOMETRICS computer program (Health Programs International, Wellesley, Mass). In addition, the 15 patients were asked to identify their most accurate pretreatment and posttreatment profile images. **Results:** Analyses of variance showed that the white orthodontists preferred flatter profiles than the black women, who in turn preferred fuller profiles than the black orthodontists. Significant differences in lip position were detected for most pleasing and midpoint of acceptability among the 3 groups of judges, with no significant differences in any variables measured between treatments in rating the 3 images. No significant differences were detected for the magnitude of the zone of acceptability. All 15 black women recalled having fuller profiles than they actually did before treatment, but they could correctly identify their own profile images after treatment. **Conclusions:** The results of this study will facilitate the understanding of the physical bases of the esthetic judgments of black and white orthodontists and black female patients. (Am J Orthod Dentofacial Orthop 2006;129:17-23)

Article 1

Table V. Descriptive statistics and results of 1-sample *t* test for MA before and after treatment

BFP	*N*	*Mean (mm)*	*SD (mm)*	P *value**
Pretreatment lower lip	15	2.507	2.886	.005*
Pretreatement upper lip	15	2.804	3.354	.006*
Posttreatment lower lip	15	0.162	1.727	.722
Posttreatment upper lip	15	–0.019	2.105	.973

*Statistically significant at $P \leq 0.05$.

Article 2

Three-dimensional assessment of mandibular advancement 1 year after surgery

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Rio de Janeiro, Brazil, and Chapel Hill, NC

Introduction: This prospective observational study evaluated changes in the 3-dimensional position and remodeling of the mandibular rami, condyles, and chin at splint removal and 1 year after mandibular advancement surgery. **Methods:** Presurgery, splint removal (4-6 weeks postsurgery), and 1-year postsurgery cone-beam computed tomography scans of 27 subjects were used. Superimposition on the cranial base was used to assess positional or remodeling changes in the anatomic regions of interest. Surface distance displacements were visually displayed and quantified by 3-dimensional color maps. A 1-sample *t* test was used to assess the average postsurgical changes of each region of interest. The level of significance was set at 0.05. **Results:** After antero-inferior chin displacement with surgery (mean, 6.81 ± 3.2 mm at splint removal), the average 1-year postsurgery displacement was not statistically significant ($P = 0.44$). Postsurgical adaptations greater than 2 mm were observed in 48% of the patients: 16% with an additional anterior-inferior displacement of the chin of 2 to 4 mm, and 4% with ≥4 mm; 20% had postero-superior movement of 2 to 4 mm, and 8% had postero-superior movement of ≥4 mm. The condyles tended to move, on average, ≤2 mm supero-posteriorly with surgery, and this small positional displacement was maintained 1 year postsurgery (right condyle, $P = 0.58$; left, $P = 0.88$). The rami exhibited outward (lateral) movements with surgery, with greater displacement of the inferior part of the rami (≥2 mm in 65% of the subjects). This torque of the ramus with surgery was stable 1 year postsurgery. **Conclusions:** Three-dimensional assessment of skeletal changes with mandibular advancement surgery shows that nearly half of the patients have >2 mm change in chin position from splint removal to the 1-year follow-up, with approximately equal chances of anterior and posterior movement. Torque of the rami usually occurs with mandibular advancement surgery. (Am J Orthod Dentofacial Orthop 2010;137:S53.e1-S53.e12)

Article 2

Table II. One-sample *t* test for the postsplint-removal changes (postsplint removal to 1 year)

Region	*Mean of difference*	*95% CI of mean*		*SD*	P *value*
Chin	– 0.40	– 1.43	0.64	2.50	0.44
Inferior ramus (right)	– 0.46	– 1.07	0.16	1.55	0.14
Inferior ramus (left)	– 0.26	– 0.98	0.46	1.82	0.46
Superior ramus (right)	– 0.31	– 0.83	0.20	1.31	0.22
Superior ramus (left)	– 0.13	– 0.66	0.41	1.35	0.63
Posterior ramus (right)	0.53	0.05	1.00	1.20	0.03*
Posterior ramus (left)	0.18	– 0.49	0.85	1.70	0.58
Condyle (right)	0.16	– 0.44	0.77	1.53	0.58
Condyle (left)	0.05	– 0.58	0.67	1.58	0.88

*$P < 0.05$ statistically significant.

Thank You

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